Heat transfer to water flowing turbulently through a rectangular duct with asymmetric heating

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Abstract-Experimental. fully established friction factors and Nusselt numbers are reported for the turbulent flow of water in a 2 : 1 rectangular duct. The measured friction factors are in good agreement with the Blasius prediction if the Reynolds number proposed by Jones is used. The measured Nusselt values are in good agreement with the results of Novotny et al. for the turbulent flow of air if the results are corrected for the effects of the Prandtl number.

The influence of asymmetric heating on the average Nusselt number was found to be much less for water as a working fluid than for air. This difference is explained by the fact that water has a higher Prandtl number than air.

1. INTRODUCTION

THE STUDY of friction and heat transfer for turbulent flow in non-circular passages, especially rectangular ducts, is important because of the wide application of these geometries to heat exchanger equipment. The knowledge of the fully developed friction factors and Nusselt numbers is important for the rational design of such systems.

A critical review of available friction factor data for fully developed turbulent flow of Newtonian fluids in rectangular channels was carried out by Jones in 1976 [l]. Jones introduced a modified Reynolds number. Re*, so defined that the fully developed, *laminar* friction factor for all aspect ratios is given by the same expression as that for the circular tube :

$$
f = 16/Re^*.
$$
 (1)

The modified Reynolds number, *Re*,* is given by the following expression

$$
Re^* = C(\rho V D_{\rm h}/\mu) \tag{2}
$$

where the value of C depends on the aspect ratio, as shown below:

Aspect ratio 1 0.75 0.50 0.23 0 c 1.125 1.106 1.029 0.878 0.667.

Jones demonstrated that the well-known predictions of the friction factor for fully established, turbulent flow in circular tubes (i.e. the Blasius or Karman-Nikuradse equations) could be used for rectangular ducts if the Reynolds number, *Re*.* is used.

The prediction of the heat transfer performance is more complicated. While the boundary conditions on the velocity are given by the relatively simple noslip condition (i.e. the velocity goes to zero on the boundaries), the thermal boundary conditions can take many forms. Furthermore. the effect of the Prandtl number must be taken into account in the case of heat transfer. In view of these considerations, it is obvious that the procedure which Jones used for friction factor will not work for heat transfer over a wide range of boundary conditions and Prandtl numbers.

A review of the literature on turbulent heat transfer in rectangular ducts revealed that most of the experimental studies involved air as the working fluid with the most common thermal boundary condition being constant heat flux on all four walls (Table 1). Several investigators imposed constant heat flux on two opposing walls, while the other two walls were not heated. With the exception of Levy *et al. [8]* who used water as a working fluid, all of the investigators using symmetrical, thermal boundary conditions report reasonable agreement with one of the commonly used circular tube correlations (i.e. Dittus-Boelter, Colburn, Sieder-Tate) where the characteristic length is taken as the hydraulic diameter.

In the case of turbulent flow through a rectangular duct with asymmetric heating, there are three experimental studies [2-4]. In addition, there are analytical predictions available for the limiting case of the parallel plate duct with asymmetric boundary conditions. Barrow [2], Hatton and Quarmby [14], Hatton *et al.* [15], Madsen [16] and Barrow and Lee [17] using different analytical models conclude that the Prandtl number has a strong effect on the Nusselt number for a fixed Reynolds number and a fixed heat flux ratio. In particular, they conclude that the effect of asymmetric heating decreased with increasing Prandtl number.

Experiments carried out with air for the case where only one side is heated were found to yield Nusselt numbers 15 to 20 lower than the corresponding Nusselt number for symmetric heating [3, 41. These experimental findings are in good agreement with the analytical predictions [2, 14-171.

NOMENCLATURE

asymmetrical heating of water flowing turbulently through a rectangular channel appear to be available. The purpose of this paper is to report such results for heat flux ratios of the opposing heated walls of 0 (i.e. one wall heated) 0.25 and 0.5.

2. EXPERIMENTAL APPARATUS AND DATA REDUCTION

The test section was a 2 : 1 rectangular duct with a hydraulic diameter of 1.20 cm and a length of 532 hydraulic diameters as shown in Fig. 1. A cross-section of the test section is presented in Fig. 2. The wider upper and lower walls were fabricated of stainless steel $(k = 15 \text{ W m}^{-1} {}^{\circ}\text{C}^{-1})$ and heating was accomplished by passing DC electric current through the steel walls. The electrical system was arranged so that each wall could be heated separately, thereby allowing both symmetric and asymmetric heating. The adiabatic side walls were fabricated of a low-conductivity plastic $(0.23 \text{ W m}^{-1} \text{K}^{-1})$. Sixteen pressure taps and 91 thermocouples were installed along the length of the duct. The rectangular duct was surrounded by a 30.5-cm plywood box filled with styrofoam beads with an effective thermal conductivity of 0.036 W m^{-1} K^{-1}.

The installed horizontal test section was positioned in a once-through flow system* consisting of a 400-

No comparable heat transfer experiments with gallon reservoir, a Moyno positive displacement pump, connecting piping, a calming section before the test section (serving as a flow straightener and establishing a uniform velocity profile at the entrance to the test section), a mixing section following the test section and finally a weighing tank. After passing through the flow loop and being weighed, the test fluid was emptied into the building drainage system.

> The following measurements were taken after reaching steady state : pressure drop ; flow rate ; voltage drop across, and current flow through, the upper and lower stainless-steel test section walls ; bulk fluid temperature at inlet and exit of the test section ; temperatures on the outside of the stainless-steel test section walls and temperatures within the plastic side walls.

> The spanwise average heat transfer coefficient, h , and the corresponding Nusselt number, Nu , were calculated at 23 axial stations along the length of the test section. Individual coefficients were calculated for the upper plate and the lower plate, based on the actual heat flux as measured by the current and voltage. The major assumptions made in the analysis of data included the neglect of axial and peripheral heat conduction which were validated by experimental measurements.

3. FRICTION FACTORS

The pressure drop measurements were taken simultaneously with heat transfer measurements. The fully developed friction factor results for all cases of symmetric and asymmetric heating are given in Fig.

^{*}A once-through system was used inasmuch as the test set-up is primarily intended for use with aqueous polymer solutions which undergo continuous degradation in a conventional circulating flow loop.

FIG. 1. Experimental set-up : flow and power supply loops.

$$
Re^* = 1.029 \rho V D_h / \mu = 1.029 Re. \tag{3}
$$

3 as a function of the modified Reynolds number The experimental results are seen to be in excellent introduced by Jones, equation (2). In the particular agreement with the Karman-Nikuradse or Blasius case of the 2: 1 aspect ratio rectangular duct, the equations which have been validated for turbulent value of C is 1.029 and the Reynolds number Re^* pipe flow. The range of uncertainty of the friction reduces to factor is of the order of 2% for Reynolds number above 10000 with the average deviation of the $P_P V D_h / \mu = 1.029Re.$ (3) measurements from the prediction being -1% .

FIG. 2. Rectangular duct cross section with details of thermocouple and pressure tap attachment (dimensions are in centimeters).

Table 1. Summary of heat transfer investigations. Turbulent heat transfer measurements-Newtonian fluids in rectangular ducts

t T_{\ast} --uniform surface temperature; q --uniform heat flux; 71111 -insulated wall.

FIG. 3. Comparison with Blasius of fully developed friction factor for turbulent flow of water in a 2 : 1 rectangular duct.

4. **HEAT TRANSFER: SYMMETRICAL HEATING**

Heat transfer and pressure drop measurements were performed with symmetrical heating of the upper and lower walls of the rectangular duct in 23 runs covering a Reynolds number range from about 6000 to 60000. To avoid end effects as well as entrance effects the local spanwise average heat transfer coefficient was calculated at the middle 16 sections $(x/D_b = 45-450)$ and the average of these values was taken as the heat transfer coefficient for the run. The standard deviations of the averaged values of the Nusselt numbers were less than 5%. The calculated uncertainty in the Nusselt number is also of the order of 5%.

The measured Nusselt numbers for the turbulent flow of water in a 2:1 rectangular duct are compared in Fig. 4 with the earlier results of Novotny et *al.* [12, 131 obtained with air in rectangular ducts having aspect ratios of $1:1,5:1$ and $10:1$. To account for the difference in Prandtl number, the ordinate is $Nu/Pr^{0.4}$ while the abscissa is the Reynolds number. The agreement between the two sets of experimental data is very good above a Reynolds number of 10 000. The Dittus-Boelter and Gnielinski equations are also shown since a recent survey by Shah and London [18] concludes that the Gnielinski equation more accurately predicts circulation pipe heat transfer performance. The Dittus-Boelter equation is in better agreement with the experimental data than the Gnielinski equation above a Reynolds number of $10⁴$. However, the slope of the experimental results is somewhat lower than the Dittus-Boelter equation and the data of Novotny are 10% lower than the prediction at a Reynolds number of 100 000.

5. **HEAT TRANSFER: ASYMMETRICAL HEATING**

To determine the influence of asymmetrical heating on the Nusselt number for water flowing turbulently in a rectangular duct with an aspect ratio of $2:1$, a series of experiments was conducted with the heating flux ratio of the two opposite stainless-steel walls being 0 (one wall heated), 0.25 and 0.50.

FIG. 4. Comparison of present results with those of Novotny et al. [12, 13] and with Dittus-Boelter and Gnielinski predictions.

A total of 18 experiments were conducted with one stainless-steel wall heated, all the other walls being adiabatic. Five of these runs were taken with the lower wall heated and the other 13 with the upper wall heated. No detectable differences were observed between the runs with the upper wall heated and those with the lower wall heated. Comparison with corresponding results for the symmetrical heating case revealed that the differences are negligible in the transition region up to a Reynolds number of 10 000. For Reynolds numbers greater than 10000 it was found that the Nusselt numbers for the case where one wall is heated are on the average about 8% lower than the Nusselt numbers for symmetrical heating.

It is interesting to compare these results with those of Sparrow et al. [3] who conducted similar experiments with air as the test fluid in a 5 : 1 aspect ratio duct. They reported that the Nusselt numbers for the case with only one wall heated were lower by about 15% than the corresponding Nusselt numbers for symmetrical heating. The fact that the current results show less influence of asymmetrical heating on the Nusselt number (8% compared with 15% found by Sparrow *et al.)* is expected because the test fluid, water, has a higher Prandtl number than the air used by Sparrow *et al.*

Eight sets of measurements were taken with both walls heated, with the upper wall heat flux equal to 25% of the lower wall heat flux. On average the calculated Nusselt values for the lower heat flux wall are higher than for the wall with the higher heat flux, the difference being about 10%. The Nusselt numbers of the less (25%) heated wall are about 4% above and for the more (100%) heated wall 6% below the values predicted by the Dittus-Boelter equation.

Fig. 5. Nusselt number vs heat flux ratio for asymmetrical heating of the two walls.

Thirteen experimental runs were carried out in a series of experiments with the upper wall heat flux equal to 50% of the lower wall heat flux. For this heating ratio the difference between the Nusselt numbers of the lower heat flux and the higher heat flux walls is smaller than in the previous cases. In general, the Nusselt numbers of the less heated wall were higher than Nusselt numbers of the more heated wall as expected.

The effects of asymmetrical heating on the Nusselt values for water flowing turbulently in a 2: 1 rectangular duct are summarized in Fig. 5 which presents the ratio of the Nusselt number for asymmetric heating to the Nusselt number for symmetric heating evaluated at a fixed Reynolds number $(Nu_r/Nu_{r=1})$ as a function of the heat flux ratio on the two heated walls q_L/q_M . The experimental results of Sparrow *et al.* and the analytical predictions of Hatton et *al.* are presented on the figure for comparison purposes. The current data for water reveal relatively little influence of asymmetric heating on the Nusselt number with the maximum departure from the symmetric heating being 8% for the case of one wall heated. It is obvious that the difference in the Prandtl number accounts for the differences between the current measurements for water $(Pr = 6.5)$ and those of Sparrow and his coworkers for air $(Pr = 0.7)$.

6. **CONCLUSIONS**

Experimental measurements of the fully established friction factors and Nusselt numbers are reported for the turbulent flow of water in a 2: 1 rectangular duct. Both symmetrical and asymmetrical thermal boundary conditions are studied. The opposing upper and lower walls of the test section were heated by passing DC electrical current through them while the narrower side walls were not heated. The thermal boundary conditions on each heated wall correspond to constant heat input per unit length and constant temperature peripherally at any axial position.

The measured, fully developed friction factors are in good agreement with the Blasius prediction if the Reynolds number proposed by Jones is used *:*

$$
f=0.079(Re^*)^{-0.25}.
$$

For the 2 : 1 duct used in this study the modified Reynolds number of Jones is only 3% higher than the conventional Reynolds number based on the hydraulic diameter. In this case the Blasius prediction based on the conventional Reynolds number is also in excellent agreement with the measurements.

The measured, fully developed Nusselt numbers for turbulent flow of water in the 2 : 1 rectangular duct with symmetric heating are in very good agreement with the results of Novotny et al. for turbulent flow of air in rectangular channels with aspect ratios of 1 : 1, 5 : 1 and 10 : 1, provided that the results are corrected for the effects of Prandtl number. Both the present results and those of Novotny are in good

agreement with the Dittus-Boelter equation above a Reynolds number of 104.

For asymmetrical heating, results are reported for values of the heat flux ratio (q_L/q_M) corresponding to 0, **0.25** and 0.50. In general, the Nusselt numbers for the lower heat flux wall were higher than those of the higher heat flux wall, with values of the Nusselt number for the symmetrical heating case falling in between. The maximum difference in the measured Nusselt numbers between the symmetrical case and the asymmetrical cases studied occur when one wall is heated and the other walls are adiabatic. This maximum difference was 8% compared to a value of 15% found by Sparrow et al. for air. This difference reflects the fact that the working fluid in the present study, water, has a higher Prandtl number than air. Over the range of q_L/q_M from 0 to 1 the local Nusselt number differs by less than 10% from the value corresponding to symmetrical heating.

Acknowledgement-The authors acknowledge the financial support of the Division of Engineering of the Office of Basic Energy Sciences of the U.S. Department of Energy under its Grant No. 85 ER 13311.

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TRANSFERT THERMIQUE A DE L'EAU EN ECOULEMENT TURBULENT DANS UN CANAL RECTANGULAIRE AVEC CHAUFFAGE DISSYMETRIQUE

Résumé-Des coefficients de frottement et des nombres de Nusselt obtenus expérimentalement sont rapportés pour l'écoulement turbulent d'eau dans un canal rectangulaire 2 : 1. Les coefficients de frottement sont en bon accord avec la prévision de Blasius si on utilise le nombre de Reynolds proposé par Jones. Les nombre de Nusselt au nombre de Froude et au nombre de Péclet. Les résultats du modèle sont comparés turbulent si les résultats sont corrigés des effets du nombre de Prandtl.

WÄRMEÜBERTRAGUNG BEI TURBULENTER WASSERSTRÖMUNG IN EINEM RECHTECKKANAL BEI ASYMMETRISCHER BEHEIZUNG

Zusammenfassung-Uber eine experimentelle Untersuchung des Widerstandsbeiwertes und der Nusselt-Zahl in einer ausgebildeten turbulenten Wasserstromung in einem Rechteckkanal (Seitenverhaltnis 2 : 1) wird berichtet. Die gemessenen Widerstandsbeiwerte stimmen gut mit Berechnungen nach Blasius überein, wenn die Reynolds-Zahl nach Jones verwendet wird. Die gemessenen Nusselt-Zahlen sind in guter Ubereinstimmung mit den Ergebnissen von Novotny für turbulente Luftströmung, wenn die Ergebnisse um die Auswirkungen der Prandtl-Zahl berichtigt werden. Der EinfluB der asymmetrischen Beheizung auf die mittlere Nusselt-Zahl ist beim Arbeitsmedium Wasser wesentlich geringer als beim Arbeitsmedium Luft. Dieser Unterschied wird durch die Tatsache erklart. da13 Wasser eine hohere Prandtl-ZahI als Luft besitzt.

TEПЛОПЕРЕНОС К ТУРБУЛЕНТНОМУ ПОТОКУ ВОДЫ, ДВИЖУЩЕМУС*Я* ЧЕРЕЗ ПРЯМОУГОЛЬНЫЙ КАНАЛ С АСИММЕТРИЧНЫМ НАГРЕВОМ

Аннотация-Представлены результаты эксперимента по определению коэффициента трения и числа Нуссельта для полностью развитого турбулентного потока воды в прямоугольном канале с ОТНОШением сторон 2:1. Измеренные значения коэффициентов трения хорошо согласуются с результатами расчета Блазиуса при использовании числа Рейнольдса, предложенного Джонсом. Измеренные значения числа Нуссельта хорошо соответствуют результатам Новотного и др. для турбулентного потока воздуха, если результаты скорректировать на молекулярное число Прандтля. Найдено, что влияние асимметричного нагрева на осредненное значение числа Нуссельта для воды является значительно меньшим, чем для воздуха. Это объясняется различием значений молекулярного числа Прандтля для воды и воздуха.